Deducing of deflection angle and expression of the θ-β-M diagram

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Abstract: The article considers one-dimensional flow, taking into account only the normal component of the velocity, which is normal to the wave. The flow deflection angle was calculated for a Mach number M = 1.5 and the CFD method was used. Then the θ-β-M diagram is expressed.

Keywords: *oblique shock wave, mass equation, momentum equation, energy equation, deflection angle*.

1. Introduction

In many of the aerodynamic tasks the shape of the nose of the aircraft is known, but the angle of formation at supersonic speeds shock wave is not known. This paper presents ways to obtain the flow deflection angle using known mathematical models.

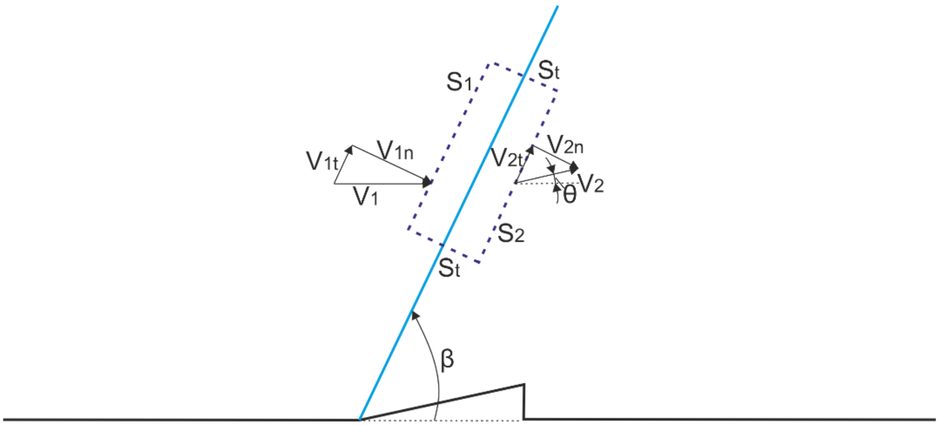

Fig.1.1. Steady conservation equations

Conservation equations describe continuity equation, motion equation and energy equation [1].

Mass equation:

$$\oiint_S \rho V dS = 0 \qquad (1.1)$$

Integral have to be expand for side before oblique shock (S_1) and after oblique shock (S_2):

$$-\rho_1 V_{1n} A_{s1} + \rho_2 V_{2n} A_{s2} = 0,$$

$$A_{s1} = A_{s2} \rightarrow \rho_1 V_{1n} = \rho_2 V_{2n} \qquad (1.2)$$

Momentum equation –2D

$$(1.3) \qquad \oiint_S V_{t\ or\ n}(\rho V dS) + \oiint_S p dS = 0$$

- tangential component

$$-(\rho_1 V_{1n} A_{s1}) V_{1t} + (\rho_2 V_{2n} A_{s2}) V_{2t} = 0$$

$$A_{s1} = A_{s2}$$

$$(1.4) \qquad \rho_1 V_{1n} = \rho_2 V_{2n}$$

$$V_{1t} = V_{2t}$$

- normal component

$$-(\rho_1 V_{1n} A_{s1}) V_{1n} + (\rho_2 V_{2n} A_{s2}) V_{2n} = -(-p_1 A_{s1} + p_2 A_{s2})$$

$$A_{s1} = A_{s2}$$

$$(1.5) \qquad p_1 + \rho_1 V_{1n}^2 = p_2 + \rho_2 V_{2n}^2$$

Energy equation –

$$(1.6) \qquad \oiint_S \rho \left(e + \frac{1}{2}V^2\right)(VdS) + \oiint_S P\,(VdS) = 0$$

$$-\rho_1 \left(e_1 + \frac{1}{2}V_1^2\right) V_{1n} A_{s1} + \rho_2 \left(e_2 + \frac{1}{2}V_2^2\right) V_{2n} A_{s2} - p_1 V_{1n} A_{s1} + p_2 V_{2n} A_{s2} = 0$$

$$-\rho_1 V_{1n} \left(e_1 + \frac{1}{2}V_1^2 + \frac{p_1}{\rho_1}\right) + \rho_2 V_{2n} \left(e_2 + \frac{1}{2}V_2^2 + \frac{p_2}{\rho_2}\right) = 0$$

$$e + \frac{p}{\rho} = h - enthalpy$$

$$\rho_1 V_{1n} \left(h_1 + \frac{1}{2}V_1^2\right) = \rho_2 V_{2n} \left(h_2 + \frac{1}{2}V_2^2\right)$$

$$\rho_1 V_{1n} = \rho_2 V_{2n}$$

$$\left(h_1 + \frac{1}{2}V_1^2\right) = \left(h_2 + \frac{1}{2}V_2^2\right)$$

$$V^2 = V_n^2 + V_t^2$$

$$V_1^2 - V_2^2 = (V_{1n}^2 + V_{1t}^2) - (V_{2n}^2 + V_{2t}^2)$$

$$V_{1t} = V_{2t}$$

$$V_1^2 - V_2^2 = V_{1n}^2 - V_{2n}^2$$

$$(1.7) \qquad h_1 + \frac{1}{2}V_{1n}^2 = h_2 + \frac{1}{2}V_{2n}^2$$

2. θ-β-M relation

For any given Mach number M_(1), there is a maximum deflection angle θmax. This is illustrated in Figure 2.1. For any given angle of deflection θ, there are two solutions for a given upstream Mach number. The smaller solution of β is called the weak shock, and the larger solution of β is the strong shock.

The obtained results for weak shock and strong shock, obtained from the presented mathematical model are presented in figure 2.1 for Mach number = 1.1, 1.5, 2.0, 2.5, 3.0.

Deflection angle θ is a function of M_1 and β [1],

$$tan\beta = \frac{V_{1n}}{V_{1t}} \qquad (1.8)$$

$$\tan(\beta - \theta) = \frac{V_{2n}}{V_{2t}}$$

$$V_{1t} = V_{2t}$$

$$\rho_1 V_{1n} = \rho_2 V_{2n}$$

$$\frac{\tan(\beta - \theta)}{tan\beta} = \frac{V_{2n}}{V_{1n}} = \frac{\rho_1}{\rho_2}$$

$$M_{1n} = M_1 sin\beta$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)M_{1n}^2}{2 + (\gamma - 1)M_{1n}^2}$$

$$\frac{\tan(\beta - \theta)}{tan\beta} = \frac{2 + (\gamma - 1)M_1^2 sin^2\beta}{(\gamma + 1)M_1^2 sin^2\beta}$$

$$tan\theta = 2cot\beta \frac{M_1^2 sin^2\beta - 1}{M_1^2(\gamma + cos2\beta) + 2} \qquad (1.9)$$

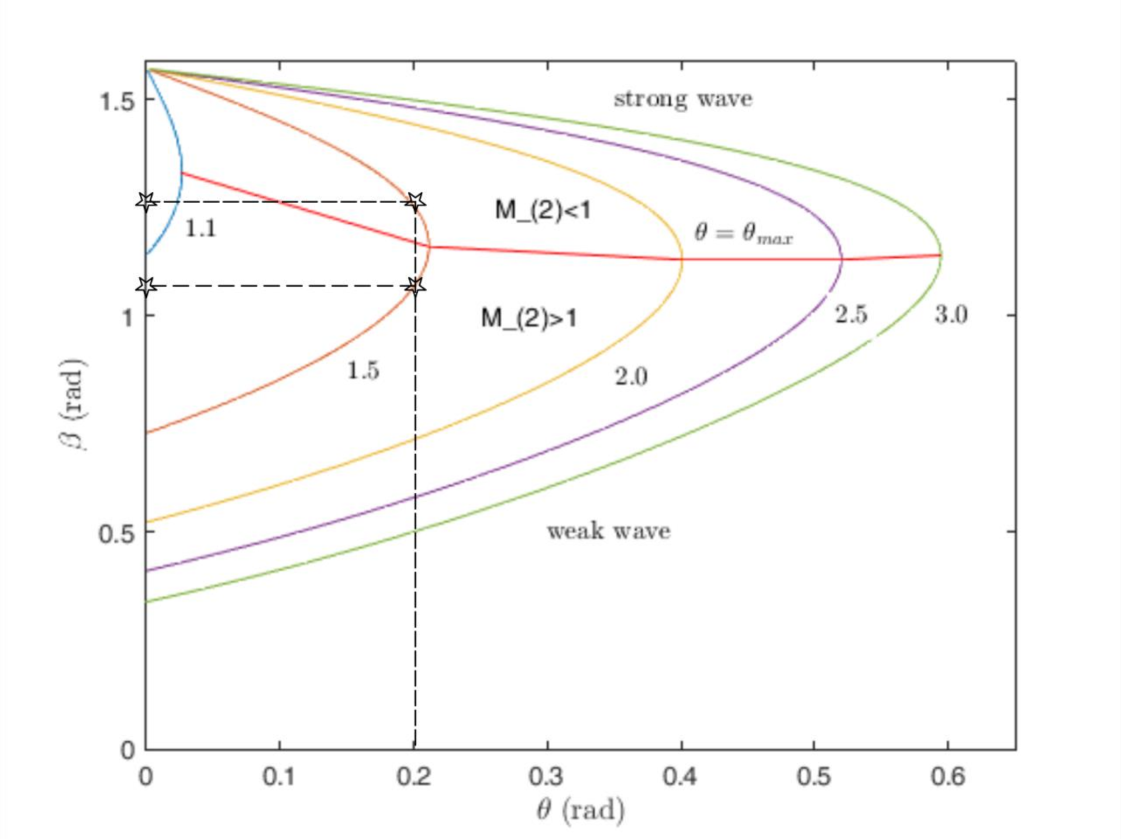

Fig. 2.1. θ-β-M relation

There are a variety of CFD products that solve tasks of different complexity. Some are limited to one-dimensional flows, others to plane flows, and still others solve spatial problems.

To solve the defined task it is necessary to perform the following:

- the area is discretized to a finite number of cells;
- general equations are discretized;
- all equations are solved and the fluid flow is obtained.

The analysis includes the creation of a mathematical model of physical phenomena:

- the laws must be in force throughout the area under consideration;
- the properties of the fluid are modeled empirically;
- assumptions are made for simplification and solution of the problem.

In Figure 2.2 an algorithm of the CFD experiment is presented.

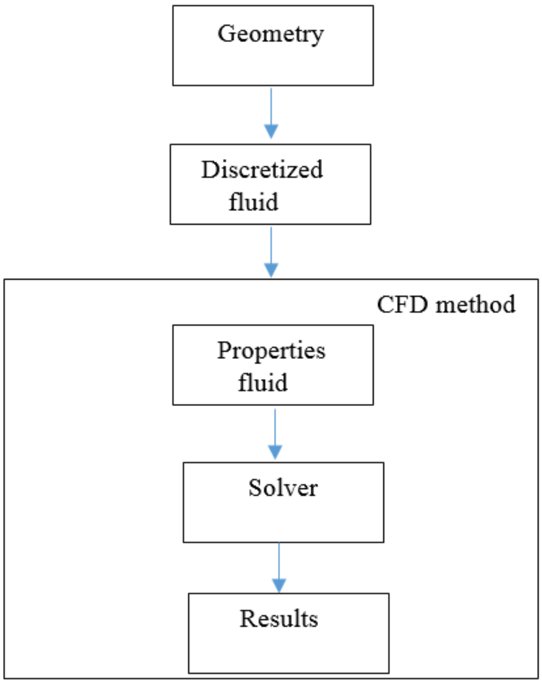

Fig. 2.2. Algorithm of CFD experiment

The result of the Mach number distribution is presented in Figure 2.3.

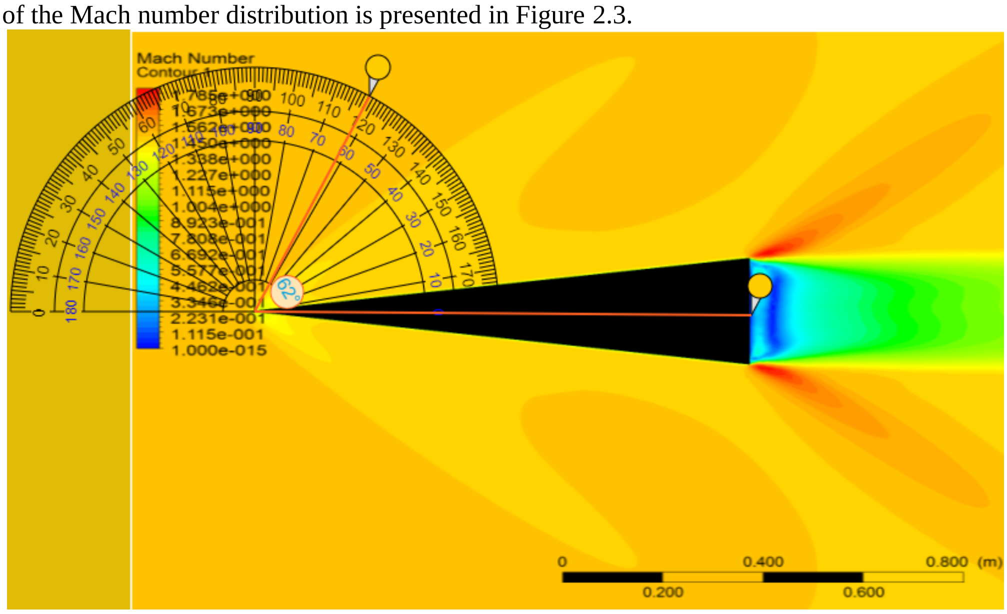

Fig. 2.3. Mach number distribution

3. Conclusion

The difference of approximately two degrees from the calculated $\theta - \beta - M$ diagram and the CFD method is due to the mesh, which is defined in the CFD method. The computational domain has coordinates 4000 2000 800, and the object in question is located in the middle with coordinates 2000 1000 400. With a maximum size of the element of the computational domain 180. If you reduce the size of the maximum element of the domain and the size of the computational cell the proximity to the flow will also reduce the difference presented.

Appendix.

```
close all; clear; clc;
gama=1.4;
beta_angle=0:0.005:(pi/2);
z=0;
for M1=[1.1,1.5,2.0,2.5,3.0]
z=z+1;
    theta_angle=atan(2*cot(beta_angle).*(((M1^2)*((sin(beta_angle)).^2))-1) ...
        ./(((gama+(cos(2*beta_angle)))*M1^2)+2));
    m(z)=max(theta_angle);
    n(z)=beta_angle(theta_angle==m(z));
    plot(theta_angle,beta_angle)
    hold on
end
```

References

1. Anderson J., Aerodynamics, McGraw-Hill Education, 2016.